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DYNAMIC PLASTIC BEHAVIOR OF STRUCTURES UNDER IMPACT LOADING INVESTIGATED BY THE EXTENDED HAMILTON'S PRINCIPLE†

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Abstract—We propose the extended Hamilton's principle to investigate the dynamic plastic behavior of a beam or a plate under an impact loading. Material is assumed to be rigid perfectly plastic. The impact loading is given in the form of initial velocity. Good agreement between our numerical solutions and experimental results done by Parkes and Jones indicates that the proposed variational method is a powerful approximation method for the dynamic plasticity analysis. The effect of strain-rate sensitivity on the permanent deflection of a plate is investigated in some detail. Instead of nonlinear differential equations, nonlinear algebraic equations are solved in the proposed method.

INTRODUCTION

Many designers have been concerned with the permanent plastic damage to structures under impact loadings such as blasts of explosion and breaking water to naval structures. For simplicity, material can be assumed to be rigid perfectly plastic in this paper. This can be justified when energy disturbance imparted by an impact load is sufficiently large and the elastic energy in the structures can be neglected. There are two approaches to solve the dynamic plastic behavior of structures subjected to impact loadings. One is infinitesimal deflection analysis where only bending moment is considered, the other is finite deflection analysis where both bending moment and membrane force are considered. The latter one is necessary when structures undergo large deflection of the order of several times the corresponding plate thickness and membrane force gives a significant contribution to the bending moment. Symonds and Mentel[1] solved analytically the dynamic plastic behavior of a beam with axial constraints. Jones[2, 3] worked out experiments on the plate models and also obtained theoretical predictions of the maximum permanent deflection of plates by solving the governing differential equation approximately.

On the other hand, Martin[4] presented some bound theorems assuming a small deflection. Martin's upper bound on the deflection is calculated in a simple manner; however, it does not give reasonable agreement with experimental results when a beam or a plate undergoes a finite deflection.

In this paper we propose the extended Hamilton's principle[5] to obtain numerical solution for the dynamic plastic behavior of a beam or a plate. Comparison between these

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numerical solutions and experimental results are made by considering the effect of its strain-rate sensitivity for the models made from mild steel.

ENERGY DISSIPATION FUNCTION

Sawczuk[6] studied the post yield behavior of a plate under static load. Membrane action due to the change in the geometry of the plate is taken into account in his study. He assumed the plastic hinge line along which an energy dissipation can take place, while the remaining part of the plate is assumed to move as a rigid body during the response. He constructed the governing equation as follows:

$$\int_A p_0 w \, \mathrm{d}A = \sum_{i=1}^{n} (M + Fw)\theta_i l_i \tag{1}$$

where A is the undeformed plate area, p_0 the external static load of the plate, w the transverse deflection and M, F are the bending moment and the membrane force, respectively. θ_i is the relative angle across the ith plastic hinge line, and l_i is the length of the ith plastic hinge line.

Equation (1) can be interpreted as an energy balance, since the left hand side of equation (1) stands for the work done by the external force, while the right hand side of equation (1) stands for the energy dissipated by bending moment and membrane force along the plastic hinge line. The term $(M + Fw)\theta$ is called the energy dissipation function per unit length of the plastic hinge line. Jones[2] modified the energy dissipation function for a dynamically loaded clamped plate as follows:

$$D = M_0(1 + 4w/H)\theta \tag{2}$$

where M_0 is the static yield moment, and H the plate thickness.

In the derivation of equation (2), the following simple yield criterion has been used

$$\begin{aligned} M/M_0 &= \pm 1, \\ F/F_0 &= \pm 1. \end{aligned} \tag{3}$$

THE EXTENDED HAMILTON'S PRINCIPLE

Hamilton's principle is well established for a conservative system. According to Courant and Hilbert[7], "The actual motion makes the value of the integral J stationary with respect to all neighboring virtual motions which lead from the initial to the final position of the system in the same interval of time," where

$$J = \int_{t_0}^{t_1} (T - U) \, \mathrm{d}t, \tag{4}$$

where T is the kinetic energy, and U the potential energy. Namely, the stationary condition expressed in equation (5), that the variation of J with respect to generalized coordinates q_i, $\dot{q}_i$ is zero, leads to Lagrange's general equation of motion.

$$\delta J = 0. \tag{5}$$

In order to apply Hamilton's principle to a nonconservative system, we propose to modify equation (5) as follows:

$$\delta J = \delta \int_{t_0}^{t_1} (T - U) \, \mathrm{d}t = \int_{t_0}^{t_1} \delta E \, \mathrm{d}t \tag{6}$$

where δE stands for the dissipated energy during dt. Equation (6) can be interpreted as an equation of energy balance. When the principle is applied to the dynamic plastic behavior of a beam or a plate, we have

$$\delta J = \delta \int_0^{t_f} \int_A (\tfrac{1}{2}\mu \dot{w}^2 + pw)\, dt = \int_0^{t_f} \int_{l_m} \overline{M}\, \delta\theta\, dl_m\, dt \tag{7}$$

where t_f is the duration of the response of a beam or a plate, μ the mass per unit area, and p the transverse pressure. $\overline{M}$ is taken as M_0 for a cantilever beam and for a clamped plate with small deflection. While $\overline{M}$ is taken from equation (2) as

$$\overline{M} = M_0\left(1 + \frac{4w}{H}\right) \tag{8}$$

for a clamped plate with large deflection. In this paper, equation (7) is the fundamental equation to predict the maximum permanent deflection due to a dynamic loading together with the initial and final conditions, i.e.

$$\begin{aligned} w = 0 \quad &\text{at} \quad t = 0 \\ \dot{w} = 0 \quad &\text{at} \quad t = t_f. \end{aligned} \tag{9}$$

The justification of the extended Hamilton's principle can be seen from the principle of energy balance as mentioned before. However, the following fact supports further justification of the principle. When the variation in the right-hand side in (7) is taken with respect to w, and the integrands with respect to the time integrals in both sides in (7) are put equal, equation (10) of [2] derived by Jones is obtained.

DYNAMIC PLASTIC BEHAVIOR OF A CANTILEVER BEAM WITH A STRIKER AT ITS TIP

Consider a cantilever beam which is suddenly struck at its tip by a moving mass N_0 with initial velocity V_0 as shown in Fig. 1. This problem was investigated by Parkes[8] experimentally and theoretically. He made the following assumptions

(i) Material is rigid perfectly plastic.
(ii) The segment between the travelling hinge and the tip undergoes rigid motion.
(iii) Velocity field of the segment is $\dot{w} = (1 - x/\xi)\dot{w}_0$.

Under these assumptions, he set up two equilibrium equations, i.e. equilibrium of force in the vertical direction and of moment about the tip. The response of a beam consists of two stages. The first stage is from $t = 0$ to $t = t_l$ until the travelling hinge reaches the built-in end, the second stage is from $t = t_l$ to $t = t_f$ until the deformation of a beam stops. He worked out the governing equations and obtained a theoretical prediction for the permanent deflection of the tip of a beam. Experimental results for hot rolled mild steel and theoretical predictions obtained by Parkes are given in Table 1.

In Parkes' experiments, two extreme cases were investigated. One is for a heavier striker, the other for a lighter striker. He found that the beam with a heavier striker deforms without the travelling hinge, as shown in Fig. 2, while the beam with a lighter striker deforms with

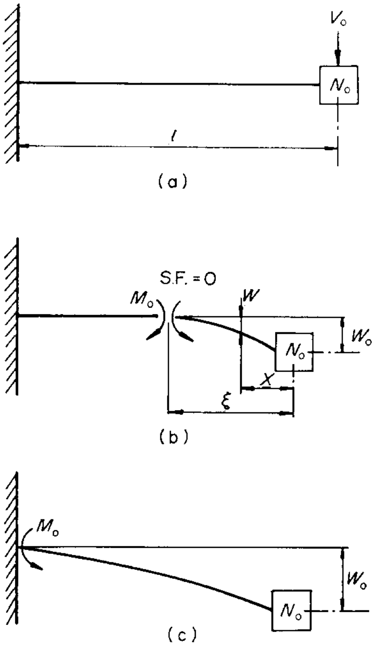


Fig. 1. A cantilever beam loaded at its tip. (a) load and dimension, (b) hinge in interior of a beam, and (c) hinge at built-in end of a beam.

Table 1. Comparison between experimental results of hot rolled mild steel and theoretical predictions of the maximum permanent deflection and the duration of response

Specimen No.	l	N_0	V_0	w_{0f} (Parkes' experiment)	w_{0f} (Parkes' theory)	w_{0f} (present theory)	w_{0f} (Martin's upper bound)	t_f (present theory)	t_f (Martin's lower bound)
1	2	4	6·4	0·22	0·21	0·22	0·22	0·56	0·56
2	4	4	6·4	0·42	0·44	0·44	0·44	1·16	1·15
3	8	4	6·4	0·93	0·93	0·95	0·94	2·47	2·44
4	12	4	6·4	1·53	1·48	1·53	1·50	3·98	3·91
5	2	4	9·0	0·43	0·42	0·43	0·42	0·79	0·79
6	4	4	9·0	0·90	0·87	0·88	0·87	1·63	1·62
7	8	4	9·0	2·15	1·85	1·88	1·86	3·48	3·44
8	12	4	9·0	3·46	2·97	3·02	3·00	5·59	5·49
9	2	1	12·7	0·18	0·21	0·21	0·21	0·28	0·27
10	4	1	12·7	0·37	0·39	0·43	0·42	0·56	0·55
11	8	1	12·7	0·77	0·85	0·88	0·85	1·17	1·11
12	12	1	12·7	1·11	1·29	1·38	1·29	1·81	1·69
13	2	1	18·0	0·37	0·42	0·42	0·42	0·39	0·39
14	4	1	18·0	0·78	0·84	0·86	0·84	0·79	0·78
15	8	1	18·0	1·68	1·70	1·78	1·70	1·65	1·57
16	12	1	18·0	2·43	2·68	2·78	2·59	2·57	2·39

N_0—the weight of the striker (lb); V_0—the initial velocity of the striker (ft/sec); l—the length of the beam (in) w_{0f}—the maximum permanent deflection (in); t_f—the duration of response (10^{-2} sec).

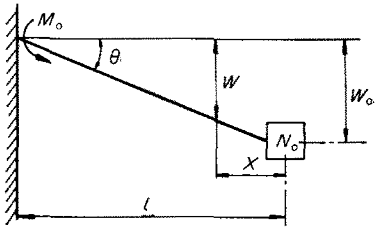


Fig. 2. Displacement field of a cantilever with a heavy striker at its tip.

the travelling hinge as shown in Fig. 1(b). In our analysis, for simplicity, we chose the case of the heavier striker. Equation (7) for this problem becomes

$$\delta\left[\int_0^{t_f} \tfrac{1}{2}N_0\dot{w}_0^2\,\mathrm{d}t + \int_0^{t_f}\int_0^l \tfrac{1}{2}m\dot{w}^2\,\mathrm{d}x\,\mathrm{d}t\right] = \int_0^{t_f} M_0\,\delta\theta\,\mathrm{d}t \tag{10}$$

when the membrane force is neglected. The displacement field along the beam is assumed as

$$w = w_0(1 - x/l) \tag{11}$$

where

$$w_0(t) = \frac{C_1}{2}(t - t_f)^2 + C_2 \tag{12}$$

with arbitrary constants C_1, C_2 which must be subjected to variation. The polynomial of (12) is the simplest form to satisfy condition $\dot{w}_0(t_f) = 0$. When the initial condition

$$\dot{w}_0(0) = V_0 \tag{13}$$

is introduced in addition to the conditions (9), we have

$$t_f = -\frac{V_0}{C_1} \tag{14}$$

$$C_2 = -\frac{C_1}{2}t_f^2. \tag{15}$$

Substituting equations (11)–(15) into equation (10), and taking variation with respect to C_1, we can determine the value of C_1. Using C_1, we obtain the duration of the response t_f and the maximum permanent deflection w_{0f} as follows:

$$t_f = \frac{V_0 l(N_0 + ml/3)}{M_0} \tag{16}$$

$$w_{0f} = \frac{V_0^2 l(N_0 + ml/3)}{2M_0}. \tag{17}$$

These theoretical predictions derived from equations (16) and (17) are given in Table 1. The agreement with the experimental results is satisfactory as shown in Fig. 3. Martin[4] derived a lower bound on the duration of the response t_f and an upper bound on the permanent deflection w_{0f}. These values are expressed as follows, when body forces are neglected.

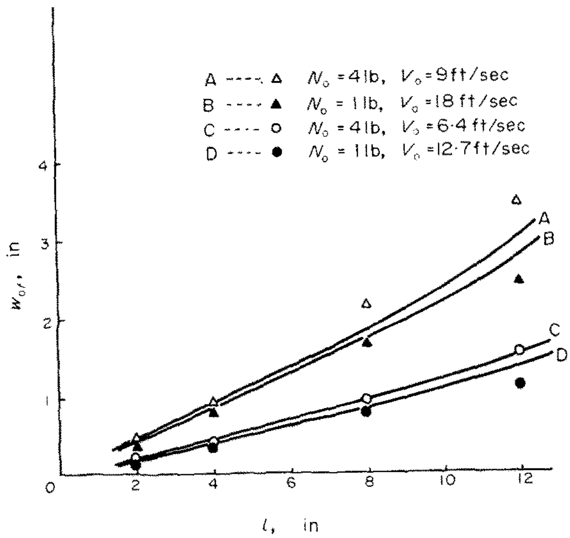


Fig. 3. Parke's experimental results of hot rolled mild steel and the present theoretical predictions (curves) for permanent deflection of a beam with a heavy striker at its tip. w_{0f} = the maximum permanent deflection, l = the length of the beam.

$$t_f \geq \frac{\int_V \rho v_i \dot{u}_i^c \, \mathrm{d}V}{D(\dot{u}_i^c)} \tag{18}$$

$$w_{0f} \leq \frac{\int_V \frac{1}{2}\rho v_i v_i \, \mathrm{d}V}{R^L} \tag{19}$$

where ρ is the mass density, v_i the initial velocity of disturbance, $\dot{u}_i^c$ the velocity of a beam, $D(\dot{u}_i^c)$ the dissipated energy associated with the assumed velocity field, and R^L is the quasi-static limit load.

If the same linear displacement field as in Fig. 2 is assumed, equations (18) and (19) become

$$t_f \geq N_0 V_0 l / M_0 \tag{20}$$

$$w_{0f} \leq N_0 V_0^2 l / 2M_0 . \tag{21}$$

These two bounds are also given in Table 1. It is observed that experimental values sometimes exceed the upper bounds. Martin's theory seems to be unsuitable for the case of large deflection, as will be discussed later.

Specimens used in Parkes' experiments were made from hot rolled mild steel which is notorious for its strain-rate sensitivity. Parkes assumed that the average strain-rate during the response of a cantilever was 1·0 per second. He therefore concluded from (50) that the dynamic yield stress becomes 1·5 times the corresponding static yield stress. The same effect of the strain-rate was employed in the present calculation.

DYNAMIC PLASTIC BEHAVIOR OF A CLAMPED RECTANGULAR PLATE WITH A UNIFORMLY DISTRIBUTED INITIAL VELOCITY

Jones[2] assumed the roof-shaped displacement field as indicated in Fig. 4. This displacement field was assured experimentally by Jones[9]. After solving the associated nonlinear

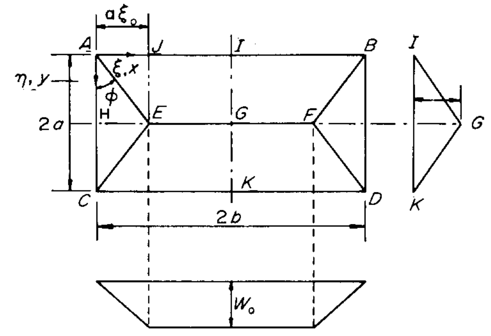


Fig. 4. Plastic hinges and coordinates.

differential equation approximately, he obtained the maximum permanent deflection as follows:

$$\frac{w_{0f}}{H} = \frac{(3 - \xi_0)\{(1 + \Gamma)^{1/2} - 1\}}{2\{1 + (\xi_0 - 1)(\xi_0 - 2)\}} \tag{22}$$

where

$$\Gamma = \frac{\lambda \beta^2}{6}(3 - 2\xi_0)\left(1 - \xi_0 + \frac{1}{2 - \xi_0}\right). \tag{23a}$$

$$\lambda = \frac{\mu V_0{}^2 a^2}{M_0 H} \tag{23b}$$

$$\beta = \frac{a}{b}, \tag{23c}$$

Jones[3] carried out experiments by using aluminum 6061T6 and hot rolled mild steel. The experimental results (triangular and rectangular marks) and his theoretical predictions (dotted curves ④) are shown in Figs. 5(a) and (b) for the case of aluminum 6061T6.

We propose to use the extended Hamilton's principle instead of solving the associated differential equations. For infinitesimal deflection, $\bar{M}$ is taken as a constant M_0, then equation (7) becomes

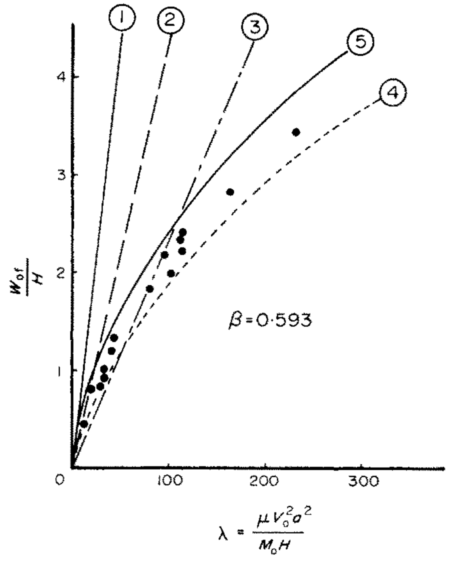
$$\delta \int_0^{t_f} \int_A \tfrac{1}{2}\mu \dot{w}^2 \, \mathrm{d}A \, \mathrm{d}t = \int_0^{t_f} \int_{l_m} M_0 \, \delta\theta \, \mathrm{d}l_m \, \mathrm{d}t. \tag{24}$$

Since $p = 0$ for $t > 0$, the second term in equation (7) has been neglected. We assume the same displacement field as indicated in Fig. 4. Calculation is carried out over a quarter part AIGH. Angle ϕ is determined from upper bound theorem of the corresponding static problem[10].

$$\tan\phi = \sqrt{(3 + \beta^2)} - \beta. \tag{25}$$

By using equation (25), it is assumed that hinge lines AH, AI, AE and EG in a quarter part are time-independent during the response of a plate under an impact loading. Central deflection of a plate w_0 is assumed as follows:

$$w_0(t) = V_0 t + d_1 t^2 \tag{26}$$



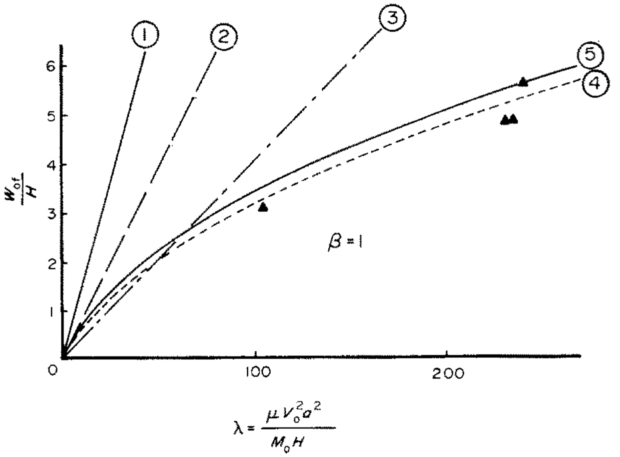


Fig. 5. Jones' experimental results of aluminum 6061 T6 and theoretical predictions (a) for $\beta = 0{\cdot}593$, and (b) for $\beta = 1$. w_{0f} = the maximum permanent deflection, H = the thickness of the plate, λ = the impulse parameter, β = the aspect ratio.
Curve ①: Infinitesimal deflection analysis in the present theory
Curve ②: Upper bound by Martin
Curve ③: Lower bound by Morales and Nevill
Curve ④: Jones' theoretical prediction
Curve ⑤: Finite deflection analysis in the present theory.

then,

$$\dot{w}_0(t) = V_0 + 2d_1 t \tag{27}$$

where V_0 is a given initial velocity and d_1 is a coefficient which is subjected to variation. The initial conditions are automatically satisfied.

$$w_0(0) = 0, \qquad \dot{w}_0(0) = V_0. \tag{28}$$

The final condition $\dot{w}_0(t_f) = 0$ gives

$$t_f = -\frac{V_0}{2d_1}. \tag{29}$$

The transverse deflection w at an arbitrary point is expressed in terms of w_0 and non-dimensional coordinates ξ, η

$$w = \begin{cases} w_0 \dfrac{\xi}{\xi_0} & 0 \le \xi \le \eta\xi_0 \quad \text{for the part } AHE \\ w_0 \eta & 0 \le \eta \le \dfrac{\xi}{\xi_0} \quad \text{for the part } AEJ \\ w_0 \eta & 0 \le \eta \le 1 \quad \text{for the part } JEGI \end{cases} \tag{30}$$

where $\xi = x/a$, $\eta = y/b$.

Using equations (27), (29) and (30), and taking variation with respect to d_1, we have

$$\delta \int_0^{t_f} \int_A \frac{1}{2} \mu \dot{w}^2 \, \mathrm{d}A \, \mathrm{d}t = \frac{7}{12} ab\mu \left(\frac{1}{3} - \frac{\xi_0}{6}\right) \frac{V_0^{\,3}}{d_1^{\,2}} \delta d_1. \tag{31}$$

From geometry, relative angle θ across a hinge line is expressed in terms of w_0

$$\theta = \begin{cases} \left(\dfrac{\cos\phi}{a\xi_0} + \dfrac{\sin\phi}{b}\right) w_0 & \text{along } AE \\ \dfrac{w_0}{b} & \text{along } AI \text{ and } EG \\ \dfrac{w_0}{a\xi_0} & \text{along } AH \end{cases} \tag{32}$$

then,

$$\int_{l_m} M_0 \, \delta\theta \, \mathrm{d}l_m = \int_{AE} M_0 \, \delta\theta \, \mathrm{d}l_m + \int_{EG+AI} M_0 \, \delta\theta \, dl_m + \int_{AH} M_0 \, \delta\theta \, \mathrm{d}l_m = M_0 k \, \delta w_0 \tag{33}$$

where

$$k = \left(\frac{\cos\phi}{a\xi_0} + \frac{\sin\phi}{b}\right) a \sqrt{(\beta^2 + \xi_0^{\,2})} + \frac{a}{b}(2 - \xi_0) + \frac{b}{a\xi_0}. \tag{34}$$

From equations (26), (29) and (33), we have

$$\int_0^{t_f} \int_{l_m} M_0 \, \delta\theta \, \mathrm{d}l_m \, \mathrm{d}t = -\frac{M_0 k V_0^{\,3}}{24 d_1^{\,3}} \delta d_1. \tag{35}$$

Thus, equation (24) leads to

$$d_1 = -\frac{M_0 k}{14\left(\dfrac{1}{3} - \dfrac{\xi_0}{6}\right) ab\mu}. \tag{36}$$

The duration of response t_f and the maximum permanent deflection w_{0f} for infinitesimal deflection analysis are obtained as follows:

$$t_f = \frac{28\left(\frac{1}{3} - \frac{\xi_0}{6}\right) V_0 ab\mu}{M_0 k} \tag{37}$$

$$w_{0f} = \frac{7}{2}\left(\frac{1}{3} - \frac{\xi_0}{6}\right) \frac{ab\mu V_0^2}{M_0 k}. \tag{38}$$

The maximum permanent deflections w_{0f} calculated from equation (38) for aluminum 6061 are shown by curves denoted by ① in Figs. 5(a) and (b). It is observed that infinitesimal deflection analysis is valid only when the magnitude of the deflection is less than about one half of the plate thickness. However the result will be much improved when the effect of membrane force is taken into account in the following way. M_0 in equation (24) will be replaced by $\overline{M}$ given in equation (8). Then, we have

$$\delta \int_0^{t_f} \int_A \tfrac{1}{2}\mu \dot{w}^2 \, \mathrm{d}A \, \mathrm{d}t = \int_0^{t_f} \int_{l_m} M_0\left(1 + \frac{4w}{H}\right) \delta\theta \, \mathrm{d}l_m \, \mathrm{d}t. \tag{39}$$

The time-dependent central deflection w_0 is assumed as before

$$w_0 = V_0 t + d_2 t^2. \tag{40}$$

Taking similar procedure as in the case of infinitesimal deflection analysis, the left hand side of equation (39) becomes

$$\frac{7}{12} ab\mu\left(\frac{1}{3} - \frac{\xi_0}{6}\right) \frac{V_0^3}{d_2^2} \delta d_2 \tag{41}$$

while the right hand side of equation (39) becomes

$$\frac{M_0 V_0^3}{d_2^3}\left\{-\frac{k_1}{24} + \frac{3k_2 V_0^2}{320 d_2 H}\right\} \delta d_2 \tag{42}$$

where

$$k_1 = \frac{2b}{a\xi_0} + \tan\phi + \frac{a}{b}(2 - \xi_0) \tag{43a}$$

$$k_2 = \frac{2b}{a\xi_0} + 2\tan\phi + \frac{4a}{b}(1 - \xi_0). \tag{43b}$$

Then equation (39) leads to

$$d_2 = -\frac{e_2 + \sqrt{(e_2^2 + 4e_1 e_3)}}{2e_1} \tag{44}$$

where

$$e_1 = \frac{7}{12}\left(\frac{1}{3} - \frac{\xi_0}{6}\right) \frac{ab\mu}{M_0} \tag{45a}$$

$$e_2 = \frac{k_1}{24} \tag{45b}$$

$$e_3 = \frac{3k_2 V_0^2}{320H} \tag{45c}$$

The final condition $t_f = -V_0/2d_2$ and $w_{0f} = w_0(t_f)$ give

$$t_f = \frac{e_1 V_0}{e_2 + \sqrt{(e_2{}^2 + 4e_1e_3)}} \tag{46}$$

and

$$w_{0f} = \frac{e_1 V_0{}^2}{2[e_2 + \sqrt{(e_2{}^2 + 4e_1e_3)}]}. \tag{47}$$

The maximum permanent deflections w_{0f} calculated from equation (47) for aluminum 6061 are shown by curves denoted by ⑤ in Figs. 5(a) and (b). Our numerical results agree with the experimental results as well as the results of Jones' which are based on the approximate solutions of the associated nonlinear differential equation. It should be mentioned that curves ④ and ⑤ are based upon the same yield criterion (3).

It is shown in reference[3] that Martin's upper bound on the maximum permanent deflection of a rigid-plastic rectangular plate with a uniformly distributed initial velocity is given as the right hand side of the following inequality.

$$\frac{w_{0f}}{H} \leq \frac{\lambda\beta^2}{12}\{\sqrt{(3+\beta^2)} - \beta\}^2 \tag{48}$$

where λ, β are defined in equations (23b) and (23c), respectively, and H is the plate thickness. In the same reference the corresponding lower bound is given by Morales and Nevill[11] as follows:

$$\frac{w_{0f}}{H} \geq \frac{\lambda\beta^2}{24}\{\sqrt{(3+\beta^2)} - \beta\}^2. \tag{49}$$

These upper and lower bounds are also plotted in Figs. 5(a) and (b) by curves ② and ③, respectively. It is seen that these two bounds deviate from the experimental results when the plate undergoes a finite deflection.

The effect of strain-rate sensitivity on the maximum permanent deflection is important for mild steel, while it is not so important for aluminum. An empirical formula to obtain a dynamic yield stress σ is given in[12] as

$$\sigma = \left[1 + \left(\frac{\dot{\varepsilon}}{D}\right)^{1/n}\right]\sigma_0 \tag{50}$$

where D and n are material constants, and σ_0 is the yield stress at lower strain-rate. The orders of D and n are $D = 40$ per sec, $n = 5$. We can guess roughly that the average strain-rate of mild steel used in Jones' experiment[3] is about 10 per sec. The average strain-rate was calculated by the deformation of a unit strip from IK to IGK (see Fig. 4) divided by the duration of response t_f. Then the dynamic yield stress becomes (from equation (50)) 1·9 times the corresponding static yield stress σ_0. In Table 2, values of the maximum permanent deflections obtained by the present theory for hot rolled mild steel are shown by using both the static yield stress and the dynamic yield stress for comparison. It is observed that the corrected theoretical predictions by using the dynamic yield stress give much better agreement with experimental results than the one by using static yield stress, especially for large deflection. The still remaining gap between the corrected theoretical predictions and the

Table 2. The effect of dynamic yield stress on the maximum permanent deflections for hot rolled mild steel

Specimen No.	β	$w_{0f}(\exp)/H$	w_{0f}^*/H	w_{0f}/H	$\lambda = \frac{\mu V_0^2 a^2}{M_0 H}$
1	0·249	0·30	0·56	0·85	74
2	0·249	0·64	1·03	1·53	189
3	0·249	0·74	1·42	2·06	315
4	0·249	1·88	2·73	3·89	984
5	0·249	1·19	1·69	2·45	426
6	0·499	0·78	1·32	1·95	88
7	0·499	1·70	1·95	2·85	167
8	0·499	2·05	2·67	3·83	286
9	0·499	3·96	4·26	6·03	663
10	0·7515	0·74	1·67	2·45	73
11	0·7515	3·10	3·00	4·30	196
12	0·7515	3·82	3·74	5·33	292
13	0·7515	6·71	6·28	8·83	747
14	0·7515	7·46	7·12	10·00	946
15	0·9996	1·58	2·00	2·93	73
16	0·9996	3·68	3·55	5·08	193
17	0·9996	4·32	4·65	6·60	315
18	0·9996	7·98	6·90	9·70	647
19	0·9996	9·31	8·90	12·46	1046

w_{0f}—the permanent deflection by using static yield stress; w_{0f}^*—the permanent deflection by using dynamic yield stress; β—a/b the aspect ratio.

experimental results for hot rolled mild steel may be caused by the fact that the average strain-rate has been taken by neglecting its change in the space and the time.

In conclusion, the application of the extended Hamilton's principle gives a simpler method to get the permanent maximum deflection and the duration of response by solving the nonlinear algebraic equations instead of solving the nonlinear differential equations.

CONCLUSION

The dynamic plastic behavior of a beam and a plate with given initial velocity is solved numerically by proposing to use the extended Hamilton's principle. It is shown that these numerical predictions agree remarkably well with experimental results for aluminum. From comparison between infinitesimal and finite deflection analysis, it follows that both bending moment and membrane force must be considered for a large deflection of order of about one half of the plate thickness and more.

It is also shown that the strain-rate sensitivity must be taken into account for hot rolled mild steel.

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Абстракт — Предлагается обобщенный принцип Гамильтона, в целью исследования динамического пластического поведения балки или пластинки, под влиянием ударной нагрузки. Принимается материал жестко- идеально пластический. Ударная нагрузка задана в форме начальной скорости. Надлежащее согласие между численными решениями авторов и экспериментальными результатами Паркеса и Джонса указывает, что предложенный вариационный метод является мощным приближенным методом для анализа динамической теории пластичности. Исследуется подробно эффект чувствительности скорости деформации, относительно постоянного прогиба пластинки. Вместо нелинейных дифференциальных уравнений, в предлагаемом методе решаются нелинейные алгебраические уравнения.